

Portfolio Optimization through Information Theory

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Bloomberg BBQ / Lightning seminar

Prices as information

Assets define probability laws

$$\frac{X(T, \omega)}{X(0)} = \frac{d\mathbb{P}^X}{d\mathbb{P}^Y}(\omega) = \frac{p_X(\omega)}{p_Y(\omega)}$$

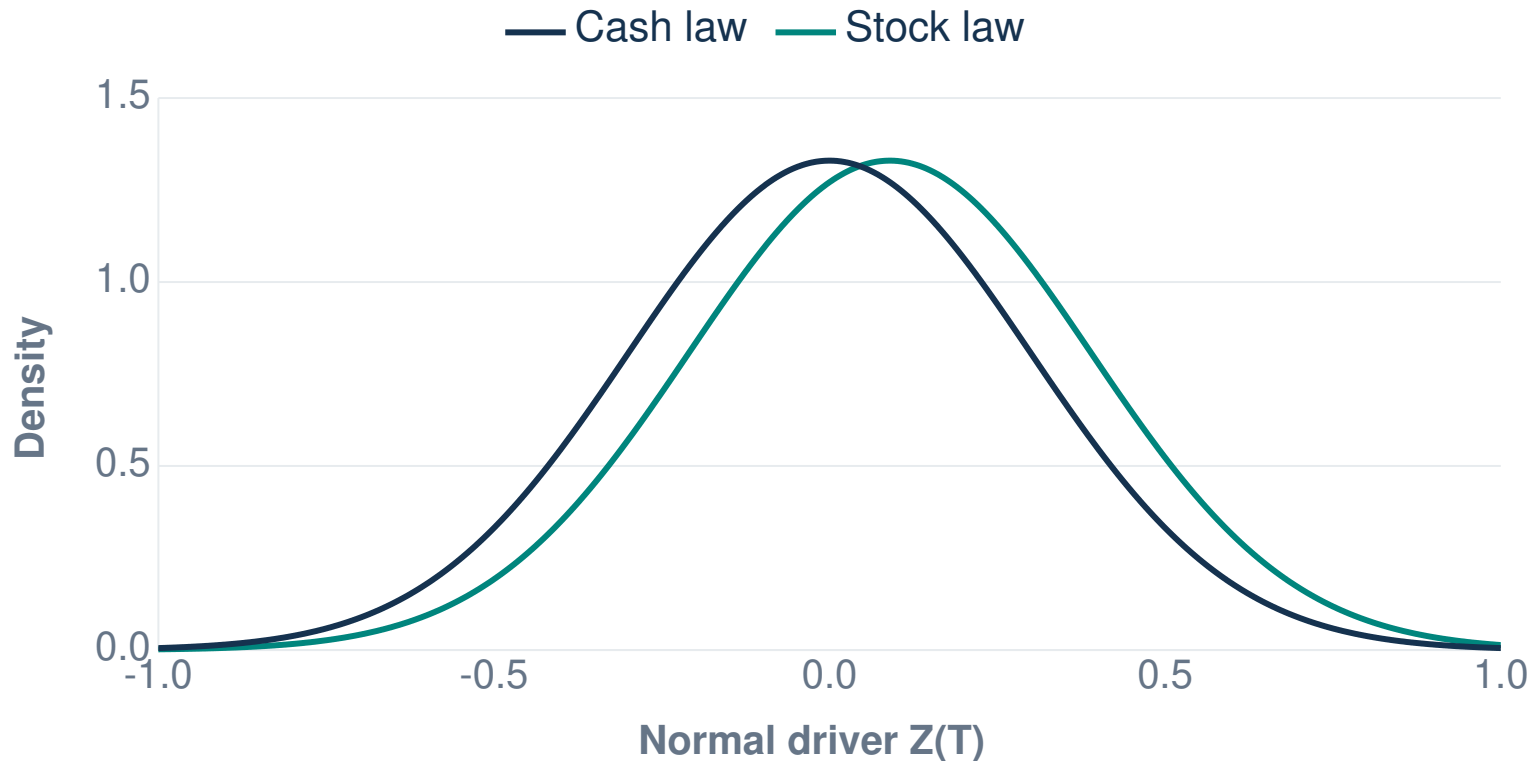
$$\mathbb{E}^X \left[\log \frac{X(T)}{X(0)} \right] = D_{\text{KL}}(\mathbb{P}^X \parallel \mathbb{P}^Y)$$

Choose which market information to retain.
Replicate the resulting payoff.

Prices are in cash-numéraire units. The normalized payoff has unit cost.

The normal example

$$\mathbb{P}^Y = \mathcal{N}(0, \sigma^2 T), \quad \mathbb{P}^X = \mathcal{N}(\sigma^2 T, \sigma^2 T)$$



$$\sigma = 0.30$$
$$T = 1 \text{ year}$$

$$D_{\text{KL}}(\mathbb{P}^X \parallel \mathbb{P}^Y) = \frac{\sigma^2 T}{2}$$

0.045

Information rate: $\sigma^2 / 2$

$$\frac{X(T)}{X(0)} = \exp\left(Z(T) - \frac{\sigma^2 T}{2}\right)$$

Four analytical linear flows

Family

Density or probability mass

KL information per unit time

Normal

$z \in \mathbb{R}$

$$p_A(z; t) = \frac{1}{\sqrt{2\pi\sigma^2 t}} \exp\left[-\frac{(z - \mu_A t)^2}{2\sigma^2 t}\right]$$

$$\frac{D(t)}{t} = \frac{\sigma^2}{2}$$

Poisson

$n = 0, 1, 2, \dots$

$$p_A(n; t) = \frac{e^{-\lambda_A t} (\lambda_A t)^n}{n!}$$

$$\frac{D(t)}{t} = \lambda_X \log \frac{\lambda_X}{\lambda_Y} - \lambda_X + \lambda_Y$$

Gamma

$z > 0$; common α

$$p_A(z; t) = \frac{\beta_A^{\alpha t} z^{\alpha t - 1} e^{-\beta_A z}}{\Gamma(\alpha t)}$$

$$\frac{D(t)}{t} = \alpha \left[\log \frac{\beta_X}{\beta_Y} + \frac{\beta_Y}{\beta_X} - 1 \right]$$

Inverse Gaussian

$z > 0$; common a

$$p_A(z; t) = \frac{at}{\sqrt{2\pi} z^{3/2}} \exp\left(ab_A t - \frac{a^2 t^2}{2z} - \frac{b_A^2 z}{2}\right)$$

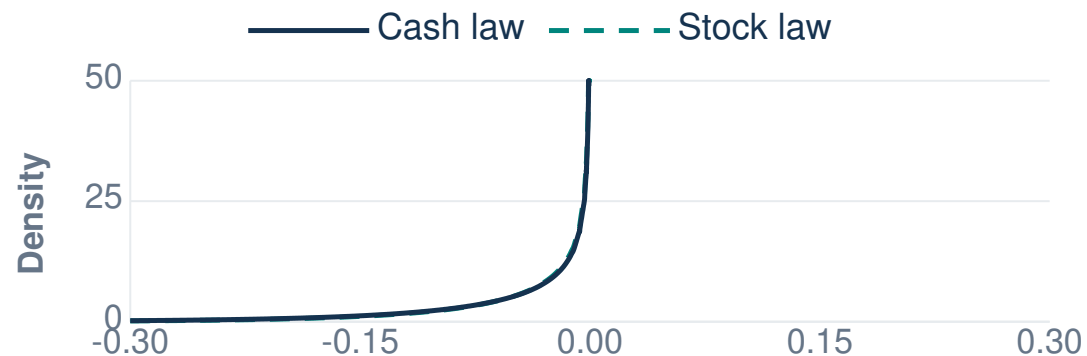
$$\frac{D(t)}{t} = \frac{a(b_X - b_Y)^2}{2b_X}$$

$$A \in \{X, Y\}, \quad \mu_Y = 0, \quad \mu_X = \sigma^2, \quad D(t) = D_{\text{KL}}(\mathbb{P}^X(t) \parallel \mathbb{P}^Y(t))$$

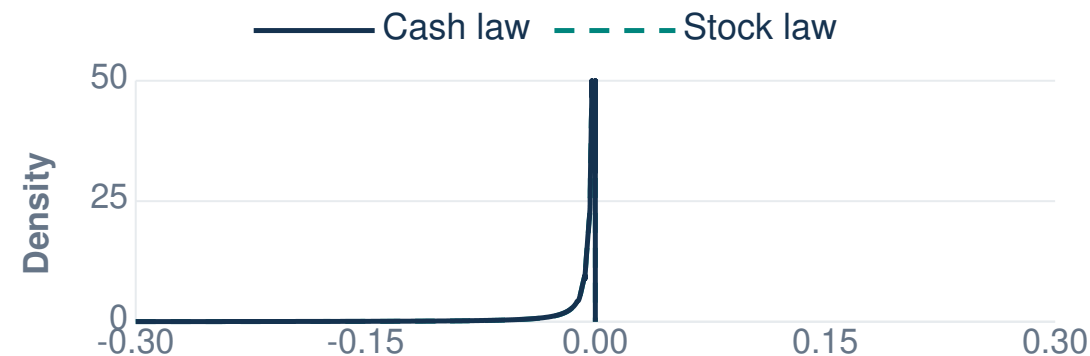
Four information channels in the S&P 500

SPX, 18 January 2019. Maturity 18 April 2019 (90 days).

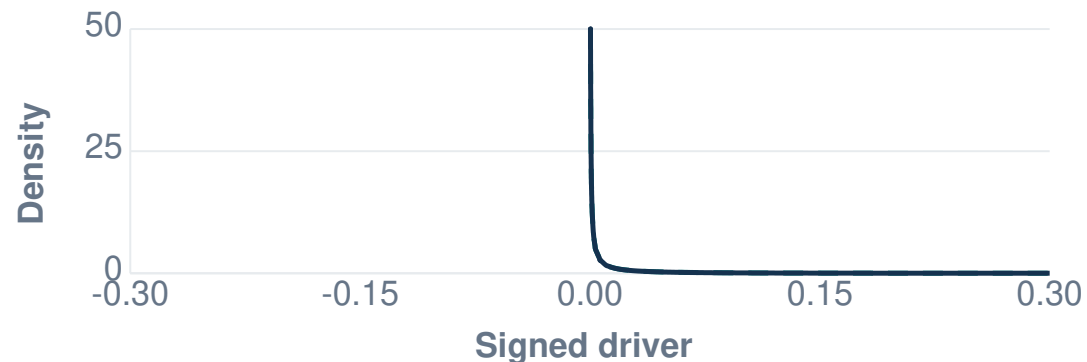
Gamma down 25.8 log bps



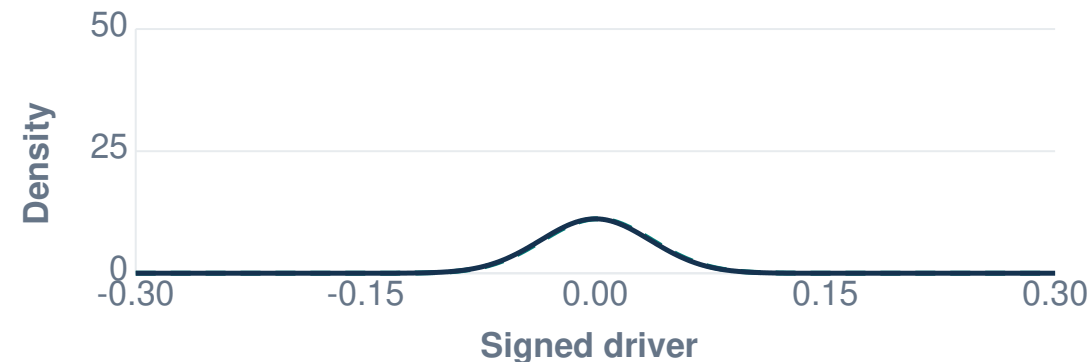
Inverse Gaussian down 7.36 log bps



Gamma up 0.977 log bps



Normal 6.42 log bps

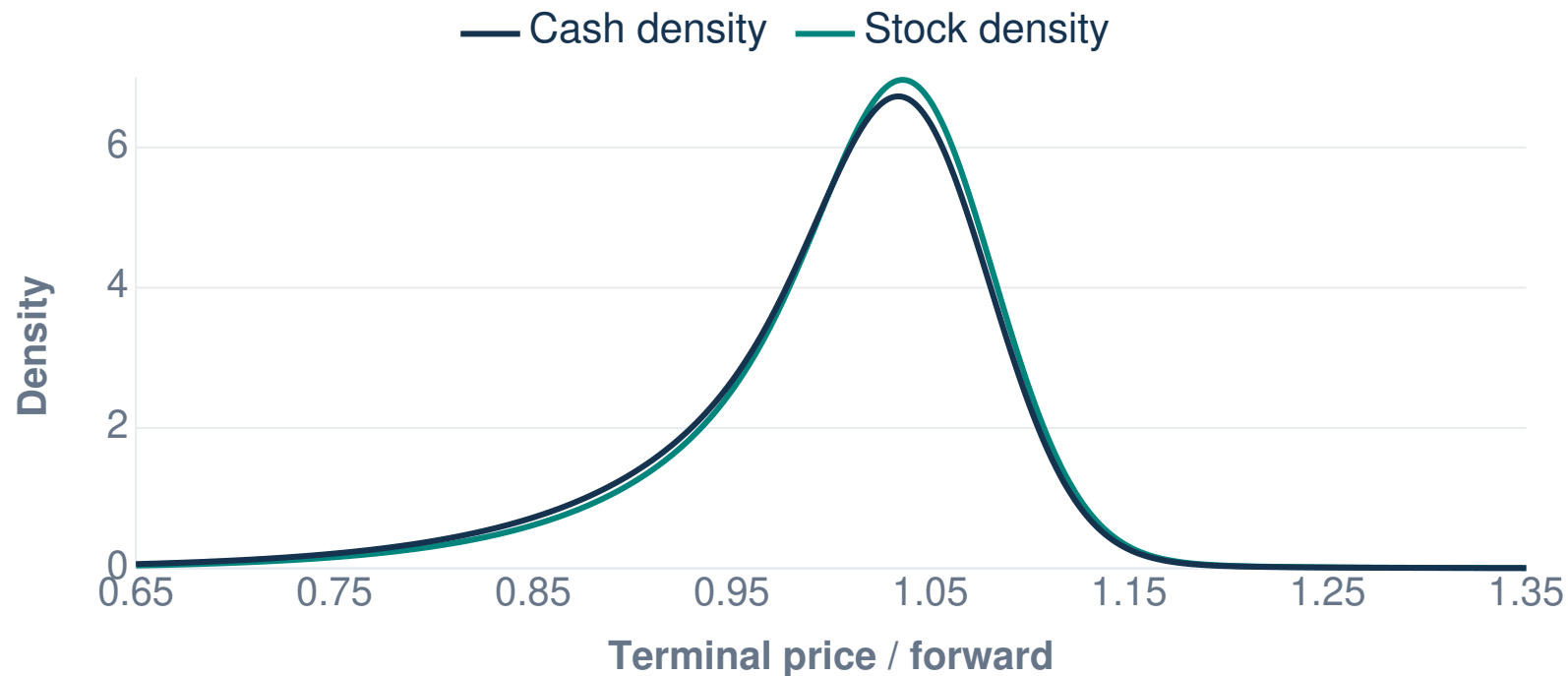


Shared linear axes. Sharp peaks near zero extend above 50. Channel split is model-dependent.

The aggregate market density

$$\log \frac{X(t)}{X(0)} = -G_{\downarrow}(t) - I_{\downarrow}(t) + G_{\uparrow}(t) + \sigma W(t) - \kappa t$$

Independent drivers. κ is the cash-law exponential compensator.



$$p_X(r) = r q(r)$$

40.5

log bps of information
over 90 days

82% comes from the
two downward channels

Three jump channels muted, normal retained

Retain the normal stock law. Choose the cash law for all three jump channels.

Jump randomness stays in the price. The selected terminal-price density is m .

Cash density: q . Normal source: 6.42 log bps. Terminal-price view: 2.44 log bps.



$$g_*(r) = \frac{m(r)}{q(r)}$$

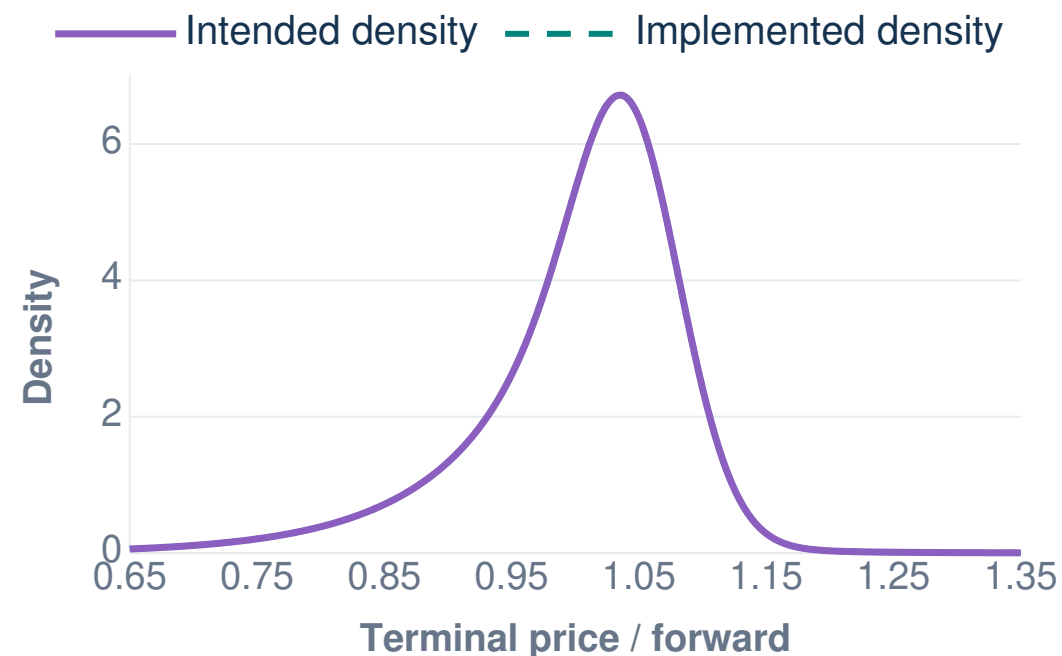
$$p_H(r) = \frac{h(r) q(r)}{\mathbb{E}^Y[h(R)]}$$

$$h_C(r) = (r - k)^+, \quad h_P(r) = (k - r)^+$$

Replication by matching densities

Nine observed strikes. Cash, stock and signed vanilla calls.

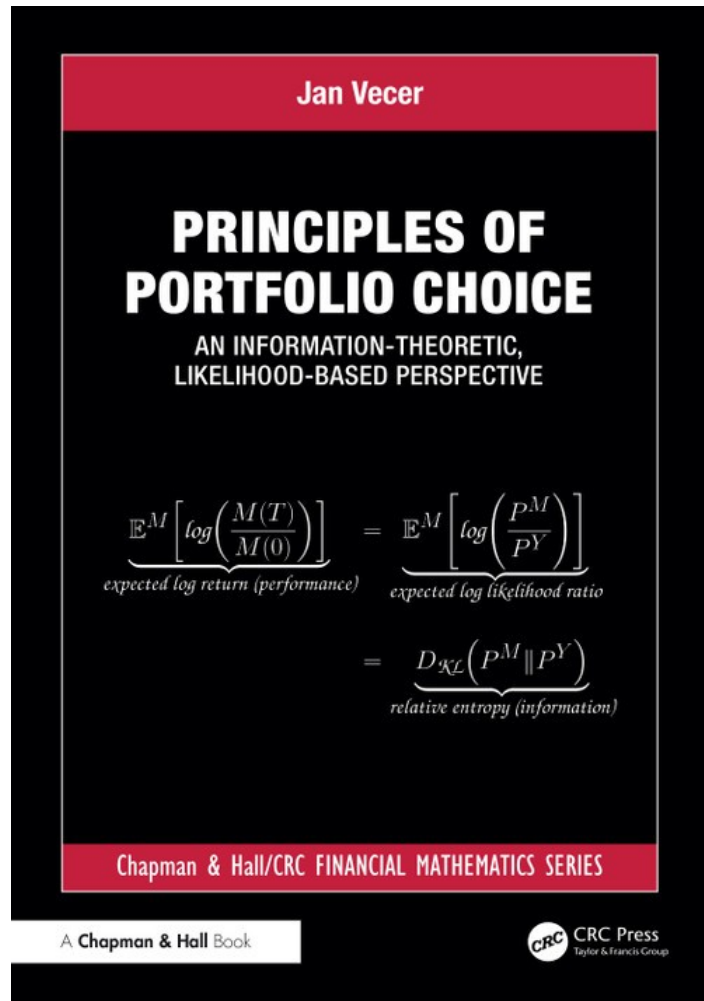
All jump channels muted. Model prices and unit initial cost.



$$\mathbb{E}^M[\log G] = D_{\text{KL}}(\mathbb{P}^M \parallel \mathbb{P}^Y) - D_{\text{KL}}(\mathbb{P}^M \parallel \mathbb{P}^G), \quad p_G = qG$$

Intended 2.44 log bps. Implemented 2.42 log bps. KL loss 0.0152 log bps.

Principles of Portfolio Choice



An Information-Theoretic,
Likelihood-Based Perspective

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